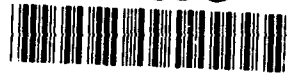


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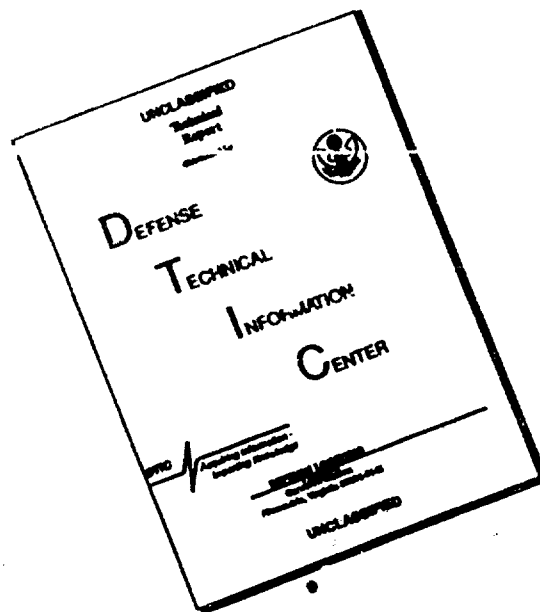
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# ANALYTICAL AND EXPERIMENTAL STUDY OF VIBRATIONS IN A GEAR TRANSMISSION

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## ABSTRACT

This paper presents an analytical simulation of the dynamics of a gear transmission system compared to experimental results from a gear noise test rig at the NASA Lewis Research Center. The analytical procedure developed couples the dynamic behaviors of the rotor-bearing-gear system with the response of the gearbox structure. The modal synthesis method is used in solving the overall dynamics of the system. Locally each rotor-gear stage is modeled as an individual rotor-bearing system using the matrix transfer technique. The dynamics of each individual rotor are coupled with other rotor stages through the nonlinear gear mesh forces and with the gearbox structure through bearing support systems. The modal characteristics of the gearbox structure are evaluated using the finite element procedure. A variable time stepping integration scheme is used to calculate the overall time transient behavior of the system in modal coordinates. The global dynamic behavior of the system is expressed in a generalized coordinate system. Transient and steady state vibrations of the gearbox system are presented in the time and frequency domains. The vibration characteristics of a simple single mesh gear noise test rig is modeled. The numerical simulations are compared to experimental data measured under typical operating conditions. The comparison of system natural frequencies, peak vibration amplitudes, and gear mesh frequencies are generally in good agreement.

## NOMENCLATURE

$A_i(t)$	modal function of the $i^{\text{th}}$ mode in x-direction
$A_{ti}(t)$	modal function of the $i^{\text{th}}$ mode in $\theta$ -direction
$B_i(t)$	modal function of the $i^{\text{th}}$ mode in y-direction
$[C_{bx}][C_{by}][C_{bz}]$	gearbox damping matrices
$[C_T]$	torsional damping matrix
$[C_{xx}], [C_{xy}], [C_{yx}], [C_{yy}]$	bearing direct and cross-coupling damping matrices
$F_{Bx}, F_{By}$	bearing excitation forces
$F_{Gr}(t)$	gear mesh torque
$F_{Gx}(t), F_{Gy}(t)$	gear mesh force in x- and y-directions

$F_T(t)$	external excitation moment
$F_x(t), F_y(t)$	external excitation forces
$[G_A]$	gyroscopic-angular acceleration matrix
$[G_V]$	gyroscopic-angular rotation matrix
$[I]$	identity matrix
$[J]$	rotational mass moment of inertia matrix
$[K]$	average stiffness matrix
$[K_{bx}][K_{by}][K_{bz}]$	gearbox stiffness matrix
$K_{dx}, K_{dy}$	compensation matrices in x- and y-direction
$K_s$	shaft stiffness matrix
$[K_T]$	torsional stiffness matrix
$K_{tik}$	gear mesh stiffness between $i^{th}$ and $k^{th}$ rotor
$[K_{xx}], [K_{xy}], [K_{yx}], [K_{yy}]$	bearing direct and cross-coupling stiffness matrix
$[M]$	mass-inertia matrix of rotor
$[M_b]$	mass-inertia matrix of gearbox
$m$	the number of modes used to define each motion
$R_{ci}$	radius of gear in the $i^{th}$ rotor
$T_F$	gear generated torque
$X, Y$	generalized motion in x- and y-directions
$X_b, Y_b, Z_b$	gearbox motion in x-, y- and z-directions
$X_{bs}, Y_{bs}$	gearbox motion at bearing supports
$X_{ci}, Y_{ci}$	gear displacements in x- and y-directions of the $i^{th}$ rotor
$X_F, Y_F$	gear forces in x- and y-directions
$X_s, Y_s$	motion of rotor at bearing support
$\alpha_{ki}$	angle of tooth mesh between $k^{th}$ and $i^{th}$ rotor
$\theta_{ci}$	rotational displacement of the $i^{th}$ rotor at the gear
$\{\theta\}_i$	rotational displacement vector of the $i^{th}$ rotor

$[A^2], [A_t^2]$	lateral and torsional eigenvalue diagonal matrices
$[\Phi]_k, [\Phi_t]_k$	lateral and torsional orthonormal eigenvector matrices of the $k^{\text{th}}$ rotor
$\phi_{kjl}$	$j^{\text{th}}$ orthonormal mode of $k^{\text{th}}$ rotor at $l^{\text{th}}$ station

## INTRODUCTION

The study of gearbox vibration is an important area of research among the engineering community. With increased speed and torque requirements and reduced allowable vibration levels, a critical need exists to develop an accurate analytical simulation of the dynamics of gear transmission systems. Recently, a number of analytical modeling and simulation procedures have been developed to predict the dynamics of multistage gear transmission systems. Some experimental work has also been performed to improve the understanding of gear stress and vibration. There exists a need to correlate analytical and experimental efforts so that the analytical methods can be refined and verified.

There is a wealth of literature on vibration analysis and simulation of gear transmission systems. August (1986) simulated the vibration characteristics of a three-planet transmission system. Pike (1987) and later Choy (1988b) used the gear relationships introduced by Cornell (1981) to study the dynamics of coupled gear systems. Mitchell (1987) applied a matrix transfer method to simulate gear vibrations. Ozguven and Houser (1988) and Kahraman (1990) used a finite element model to predict the dynamics of a multigear mesh system. Choy (1991) applied a modal synthesis technique in conjunction with finite element and the matrix transfer methods in both time and frequency domains to calculate the transient rotor and casing motions.

There are few studies correlating analytical predictions and experimental results. Some experimental correlation work by Lim (1990) compared analytical predictions of vibration of a gear housing and mounting with measurements.

This paper reviews the development of a global dynamic model of a gear train system, and presents results of a comparison made between the analytical predictions of the model and experimental results from a gear test rig. A combined approach using the modal synthesis and finite element method for analyzing the dynamics of gear systems was developed. The method includes dynamic coupling between the housing and the gear-rotor system. The single mesh gear noise test rig at NASA Lewis was used as an example in this analysis. The gearbox modal characteristics were evaluated by solving a finite element model. These modal characteristics were compared to those measured in the experimental study to insure accurate modeling of the experimental rotor-gear-housing system. Transient vibrations of the rotor-gear stages and the gearbox housing were analytically evaluated by the modal method. A numerical FFT (Fast Fourier Transform) algorithm was used to examine vibration in the frequency domain. Frequency spectra of the predicted gearbox vibration were compared to those measured in the experimental study. The following discussion and conclusions are drawn from these comparisons between the experimental results and analytical predictions.



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## ANALYTICAL PROCEDURE

The equations of motion for a single mesh multimass rotor-bearing-gear shaft system, with the coupling effects of gearbox vibrations and the rotor inertia-gyroscopic effects, can be written in matrix form for the  $i^{\text{th}}$  rotor (Choy 1987, 1989) for the X-Z plane as:

$$\begin{aligned} [M]_i \{\ddot{X}\}_i + [G_v]_i \{\dot{Y}\}_i + [C_{xx}]_i \{\dot{X} - \dot{X}_b\}_i + [C_{xy}]_i \{\dot{Y} - \dot{Y}_b\}_i + [G_A]_i \{Y\}_i + [K_{xx} + K_s]_i \{X\}_i \\ - [K_{xx}]_i \{X_{bs}\}_i + [K_{xy}]_i \{Y - Y_{bs}\}_i = \{F_x(t)\}_i + \{F_{Gx}(t)\}_i \end{aligned} \quad (1)$$

and in the Y-Z plane as:

$$\begin{aligned} [M]_i \{\ddot{Y}\}_i - [G_v]_i \{\dot{X}\}_i + [C_{yx}]_i \{\dot{X} - \dot{X}_b\}_i + [C_{yy}]_i \{\dot{Y} - \dot{Y}_b\}_i - [G_A]_i \{X\}_i + [K_{yy} + K_s]_i \\ \times \{Y\}_i - [K_{yy}]_i \{Y_{bs}\}_i + [K_{yx}]_i \{X - X_{bs}\}_i = \{F_y(t)\}_i + \{F_{Gy}(t)\}_i \end{aligned} \quad (2)$$

Here  $F_x$  and  $F_y$  are force excitations from the effects of mass imbalance and shaft residual bow in the X- and Y-directions.  $F_{Gx}$  and  $F_{Gy}$  are the X and Y gear mesh forces induced from gear teeth interactions. The bearing forces are evaluated from the relative motion between the rotor  $\{X\}$ ,  $\{Y\}$  and the gearbox  $\{X_b\}$ ,  $\{Y_b\}$  at the bearing locations (Choy 1987). The mass-inertia and gyroscopic effects are incorporated in the mass matrix  $[M]$  and the gyroscopic matrices  $[G_v]$  and  $[G_A]$ . The coupled torsional equations of motion for the single gear-rotor system can be written as:

$$[J]_i \{\ddot{\theta}\}_i + [C_T]_i \{\dot{\theta}\}_i + [K_T]_i \{\theta\}_i = \{F_T(t)\}_i + \{F_{Gt}(t)\}_i \quad (3)$$

In Eq. (3),  $\{F_T(t)\}$  represents the externally applied torque and  $\{F_{Gt}(t)\}$  represents the gear mesh induced moment. Note that Eqs. (1) to (3) repeat for each single gear-rotor system. The gear mesh forces couple the force equations of each gear-rotor system to each other as well as the torsional equations to the lateral equations (Choy, 1989; Cornell, 1981; David, 1987 and 1988). In addition, there are equations of motion for the gearbox which couple the various gear-shaft-systems through the bearing supports. The gearbox equations can be written as:

X-equation

$$\begin{aligned} [M_b] \{\ddot{X}_b\} + [C_{bx}] \{\dot{X}_b\} + [C_{xx}] \{\dot{X}_b - \dot{X}_s\} + [C_{xy}] \{\dot{Y}_b - \dot{Y}_s\} + [K_{bx}] \{X_b\} \\ - [K_{xx}] \{X_b - X_s\} + [K_{xy}] \{Y_b - Y_s\} = 0 \end{aligned} \quad (4)$$

Y-equation

$$\begin{aligned}
& [M_b] \{\ddot{Y}_b\} + [C_{by}] \{\dot{Y}_b\} + [C_{yx}] \{\dot{X}_b - \dot{X}_s\} + [C_{yy}] \{\dot{Y}_b - \dot{Y}_s\} + [K_{by}] \{Y_b\} \\
& - [K_{yy}] \{Y_b - Y_s\} + [K_{yx}] \{X_b - X_s\} = 0
\end{aligned} \tag{5}$$

and Z-equation

$$[M_b] \{\ddot{Z}_b\} + [C_{bz}] \{\dot{Z}_b\} + [K_{bz}] \{Z_b\} = F_{bz}(t) \tag{6}$$

where  $F_{bz}(t)$  is an excitation function due to external forces in the axial direction. Since the bearing is assumed to be uncoupled in the Z-direction, Eq. (6) can be solved independently without considering shaft motion.

The torsional and lateral vibrations of a single individual rotor and the dynamic relationships between each gear-rotor-system are coupled through the nonlinear interactions in the gear mesh. The gear mesh stiffness varies in a periodic nonlinear pattern with each tooth pass engagement period (August, 1986; Cornell, 1981; Savage, 1986) and can be represented by a high order polynomial (Cornell, 1981; Boyd, 1987). For the coordinate system as shown Fig. 1, the following gear mesh coupling equations are established by equating forces and moments (Choy, 1989). For the  $k^{th}$  gear of the system, summing forces in the X-direction results in:

$$F_{Gxk} = \sum_{i=1, i \neq k}^n K_{tki} \left[ -R_{ci} \theta_{ci} - R_{ck} \theta_{ck} + (X_{ci} - X_{ck}) \cos \alpha_{ki} + (Y_{ci} - Y_{ck}) \sin \alpha_{ki} \right] \cos \alpha_{ki} \tag{7}$$

Summing forces in the Y-direction results in:

$$F_{Gyk} = \sum_{i=1, i \neq k}^n K_{tki} \left[ -R_{ci} \theta_{ci} - R_{ck} \theta_{ck} + (X_{ci} - X_{ck}) \cos \alpha_{ki} + (Y_{ci} - Y_{ck}) \sin \alpha_{ki} \right] \sin \alpha_{ki} \tag{8}$$

Summing moments in the Z-direction results in:

$$F_{Gzk} = \sum_{i=1, i \neq k}^n R_{ck} \left\{ K_{tki} \left[ (-R_{ci} \theta_{ci} - R_{ck} \theta_{ck}) + (X_{ci} - X_{ck}) \cos \alpha_{ki} + (Y_{ci} - Y_{ck}) \sin \alpha_{ki} \right] \right\} \tag{9}$$

where  $n$  is the number of gears in the system.

Using the modal expansion approach (Choy, 1987, 1988a, 1989, and 1990), the motion of the system can be expressed as:

$$\begin{aligned}
\{X\} &= \sum_{i=1}^m A_i \{\phi_i\}, \{X_b\} = \sum_{i=1}^m A_{bi} \{\phi_{bx1}\} \\
\{Y\} &= \sum_{i=1}^m B_i \{\phi_i\}, \{Y_b\} = \sum_{i=1}^m B_{bi} \{\phi_{by1}\} \\
\{\theta\} &= \sum_{i=1}^m A_{ti} \{\phi_{ti}\}, \{Z_b\} = \sum_{i=1}^m D_{bi} \{\phi_{zi}\}
\end{aligned} \tag{10}$$

where  $m$  is the number of modes used to define each motion.

Using modal expansion and the orthogonality conditions, (Choy 1989, 1990) with the bearing forces due to the base motion expressed on the right hand side of the equations the modal equations of motion for the gear-rotor system (Choy 1989) can be written as:

X-Z equation

$$\begin{aligned}
\{\ddot{A}\} + [\Phi]^T [G_v] [\Phi] \{\dot{B}\} + [\Phi]^T [C_{xx}] [\Phi] \{\dot{A}\} + [\Phi]^T [C_{xy}] [\Phi] \{\dot{B}\} + [\Phi]^T [G_A] [\Phi] \{B\} \\
+ [\Lambda^2] \{A\} + [\Phi]^T [K_{dx}] [\Phi] \{A\} + [\Phi]^T [K_{xy}] [\Phi] \{B\} = [\Phi]^T \{F_x(t) + F_{Gx}(t) + F_{Bx}(t)\}
\end{aligned} \tag{11}$$

which can also be expanded into modal parameters as where

$$F_{Bx}(t) = [C_{xx}] [\Phi_{bx}] \{\dot{A}_b\} + [C_{xy}] [\Phi_{by}] \{\dot{B}_b\} + [K_{xx}] [\Phi_{bx}] \{A_b\} + [K_{xy}] [\Phi_{by}] \{B_b\} \tag{12}$$

and

Y-Z equation

$$\begin{aligned}
\{\ddot{B}\} - [\Phi]^T [G_v] [\Phi] \{\dot{A}\} + [\Phi]^T [C_{yx}] [\Phi] \{\dot{A}\} + [\Phi]^T [C_{yy}] [\Phi] \{\dot{B}\} - [\Phi]^T [G_A] [\Phi] \{A\} \\
+ [\Lambda^2] \{B\} + [\Phi]^T [K_{dy}] [\Phi] \{B\} + [\Phi]^T [K_{yx}] [\Phi] \{A\} = [\Phi]^T \{F_y(t) + F_{Gy}(t) + F_{By}(t)\}
\end{aligned} \tag{13}$$

where

$$F_{By}(t) = [C_{yx}] [\Phi_{bx}] \{\dot{A}\} + [C_{yy}] [\Phi_{by}] \{\dot{B}\} + [K_{yx}] [\Phi_{bx}] \{A\} + [K_{yy}] [\Phi_{by}] \{B\} \tag{14}$$

and the  $\theta$ -equation can be expressed as

$$\{\ddot{A}_t\} + [\Phi_t]^T [C_t] [\Phi_t] \{\dot{A}_t\} + [\Lambda_t^2] \{A_t\} = [\Phi_t]^T \{F_t(t) + F_{Gt}(t)\} \tag{15}$$



The gear mesh forces and moments can also be expressed in the modal form, for the  $k^{\text{th}}$  gear with the gear location at the  $l^{\text{th}}$  node, as:

$$[\Phi]_k^T \{F_{Gx}\} = \sum_{j=1}^m \phi_{kjl} \left\{ \sum_{i=1, i \neq k}^n K_{tki} \left[ -R_{ci} \theta_{ci} - R_{ck} \theta_{ck} + (X_{ci} - X_{ck}) \cos \alpha_{ki} \right. \right. \\ \left. \left. + (Y_{ci} - Y_{ck}) \sin \alpha_{ki} \right] \cos \alpha_{ki} \right\} \quad (16)$$

$$[\Phi]_k^T \{F_{Gy}\} = \sum_{j=1}^m \phi_{kjl} \left\{ \sum_{i=1, i \neq k}^n K_{tki} \left[ -R_{ci} \theta_{ci} - R_{ck} \theta_{ck} + (X_{ci} - X_{ck}) \cos \alpha_{ki} \right. \right. \\ \left. \left. + (Y_{ci} - Y_{ck}) \sin \alpha_{ki} \right] \sin \alpha_{ki} \right\} \quad (17)$$

$$[\Phi]_k^T \{F_{Gt}\} = \sum_{j=1}^m \phi_{kjl} \left\{ \sum_{i=1, i \neq k}^n R_{ck} \left\{ K_{tki} \left[ (-R_{ci} \theta_{ci} - R_{ck} \theta_{ck}) + (X_{ci} - X_{ck}) \cos \alpha_{ki} \right. \right. \right. \\ \left. \left. \left. + (Y_{ci} - Y_{ck}) \sin \alpha_{ki} \right] \right\} \right\} \quad (18)$$

where  $k$  is the gear-shaft-system number,  $j$  is the mode number, and  $l$  is the station location of the gear mesh.

A set of modal equations of motion can also be written for the gearbox (Choy 1989, 1990) as:

$$[I] \{\ddot{A}_b\} + [C_{bx}] \{\dot{A}_b\} + [A_b^2] \{A_b\} + [\Phi_{bx}]^T \{F_{Bx}(t)\} - [\Phi_{bx}]^T \{ [C_{xx}] [\Phi] \{\dot{A}\} + [C_{xy}] [\Phi] \{\dot{B}\} \\ + [K_{xx}] [\Phi] \{A\} + [K_{xy}] [\Phi] \{B\} \} = 0 \quad (19)$$

Similarly, the Y-equation can be written as

$$[I] \{\ddot{B}_b\} + [C_{by}] \{\dot{B}_b\} + [A_b^2] \{B_b\} + [\Phi_{by}]^T \{F_{By}(t)\} - [\Phi_{by}]^T \{ [C_{yx}] [\Phi] \{\dot{A}\} + [C_{yy}] [\Phi] \{\dot{B}\} \\ + [K_{yx}] [\Phi] \{A\} + [K_{yy}] [\Phi] \{B\} \} = 0 \quad (20)$$

and the Z-equation as

$$[I]\{\ddot{D}_b\} + [C_{bz}]\{\dot{D}_b\} + [A^2]\{D_b\} = [\Phi_{bz}]^T \{F_{bz}(t)\} \quad (21)$$

The procedure involves the solution of the coupled modal equations of motion between the (gear) rotors and the gearbox structure. The coupling effects of the gear mesh and the bearing supports are also expressed in modal coordinates such that the global equations are solved simultaneously in modal form. A set of initial conditions for both displacement and velocity of the global system are calculated from the steady state conditions at the rotor-bearing systems and zero vibration at the gearbox. The modal accelerations  $A$ ,  $B$ ,  $A_t$ ,  $A_b$ ,  $B_t$ , and  $D_b$  of the system can be evaluated (Eqs. (11), (13), (15), and (19) to (21)). A variable time stepping integration scheme (the Newmark-Beta Method) is used to integrate the modal acceleration to evaluate the modal velocities and displacements at the next time step. A regular time interval of 200 points per shaft revolution is used in this study except for a refined region of smaller steps at the transition from single to multiple tooth contact. The modal acceleration, velocity, and displacement calculated from the transient integration scheme are transformed back into the generalized coordinates (Eq. (10)). The nonlinear bearing forces and gear mesh forces can be evaluated from the velocity and displacement differentials between the rotors and the gearbox structure.

#### EXPERIMENTAL STUDY

The gear noise rig (Fig. 2) was used to measure the vibration, dynamic load, and noise of a geared transmission. The rig features a simple gearbox (Fig. 3) containing a pair of parallel axis gears supported by rolling element bearings. A 150-kW (200-hp) variable-speed electric motor powers the rig at one end, and an eddy-current dynamometer applies power-absorbing torque at the other end. The test gear parameters are shown in Table I.

Two phases of experiments were performed on the gearbox; (1) static modal analysis of the gearbox and (2) dynamic vibration measurements during operation. Modal parameters, such as system natural frequencies and their corresponding mode shapes, were obtained through transfer function measurements using a two channel dynamic signal analyzer and modal analysis software. For this experiment, 116 nodal points (with 3° of freedom each) were selected on the gearbox housing. Vibration data was collected from accelerometers placed on the gearbox at a few of the node points used in the modal survey. Water-fall plots of frequency spectra from node 21 on the gearbox top and from node 40 on the gearbox end are presented in Figs. 4 and 5. The modal frequencies (from Figs. 6 and 7) and the gear mesh frequency are shown on figures 4 and 5.

#### DISCUSSION

The experimentally obtained modes, shown in Figs. 6 and 7, represent the major vibration modes of the gear noise rig in the 0 to 3 kHz region. Although these modal frequencies are only a small part of the total modes of

the system, they represent a major part of the total global vibration of the system. In order to produce a compatible analytical simulation of the test apparatus, a similar set of modes were generated using a finite element model of the gearbox structure. Out of a total of 25 modes existing in the analytical model in the 0 to 3 kHz frequency region, the eight dominant modes were used to represent the gearbox dynamic characteristics. These analytically simulated modes are shown in Figs. 8 and 9. As shown in Table II, the natural frequencies of the simulated modes are within 5 percent of the measured modes. The three-dimensional analytical mode shapes are very similar to the experimental modes shapes (Figs. 6 and 7). The correlation in the results between the analytical model and the experimental measurements confirms the accuracy of the dynamic representation of the test gearbox using only a limited amount of modes.

In the experimental vibration spectra of Fig. 4 (from the gearbox top), there are prominent peaks at the gear mesh frequency on the traces on 1500 rpm (700 Hz), 3750 rpm (1750 Hz), and 5500 rpm (2567 Hz). These frequencies are near natural frequencies found in the modal survey (658, 1762, and 2536 Hz, respectively). At the highest speed (5750 rpm), the higher modes (2536, 2722, and 2962 Hz) are excited by the gear mesh vibration and its sidebands. Similar behavior is shown in the vibration spectra from the side of the gearbox (Fig. 5). In both of these figures, the amplitude of the peaks increases with speed as the mass imbalance force increases with the square of the rotational speed.

The vibration response of the gearbox was simulated by the analytical method presented in section II. The gearbox vibration modes were calculated by finite element methods. The gear-rotor system response was calculated with a discrete rotor model using the matrix transfer technique. The gear-rotor system was coupled to the gearbox in modal coordinates to solve for the transient vibration response of the global system. The vibration response was then transformed into the frequency domain through a fast Fourier transformation routine.

The analytical vibration spectra shown in Figs. 10 and 11 simulate the experimental results of Figs. 4 and 5. Both the experimental spectra (Figs. 4 and 5) and the analytical spectra (Figs. 4 and 5) show excitation at the natural frequencies at shaft speeds of 1500, 3500, and 5500 rpm. At the higher shaft speed (5500 rpm), several modes are excited by the shaft frequency and its sidebands. Small differences between the analytical and experimental spectra may be due to (1) small errors (<5 percent) in the calculated modes, (2) nonuniformities in the gearbox not present in the modal, (3) errors in measurement of rotational speeds and vibration amplitudes, and (4) limitations in modeling the bearings as a radial stiffness element only.

## CONCLUSIONS

Analytical and experimental studies of a single stage gear transmission system were performed. Results from the analytical simulations show good correlation with experimentally obtained data from the test gearbox. Some basic conclusions from this study can be summarized as follows:

1. A modal synthesis approach can be used to simulate the dynamics of a gear transmission system.

2. The accurate gearbox model can be developed by correlating the modal characteristics from experimental study with those from analytical simulations.

3. The proper choice of modes used in the modal synthesis will reduce the number of modes required in the analysis, without sacrificing accuracy in the numerical solution.

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TABLE I. - TEST GEAR PARAMETERS

Gear type . . . . .	Standard involute, full-depth tooth
Number teeth . . . . .	28
Module, mm (diametral pitch in. <sup>-1</sup> ) . . . . .	3.174(8)
Face width, mm (in.) . . . . .	6.35(0.25)
Pressure angle, deg . . . . .	20
Theoretical contact ratio . . . . .	1.64
Driver modification amount, mm (in.) . . . . .	0.023(0.0009)
Driven modification amount, mm (in.) . . . . .	0.025(0.0010)
Driver modification start, deg . . . . .	24
Driven modification start, deg . . . . .	24
Tooth-root radius, mm (in.) . . . . .	1.35(0.053)
Gear quality . . . . .	AGMA class 13
Nominal (100 percent) torque, N-m(in.-lb) . . . . .	71.77(635.25)

TABLE II. - COMPARISON OF EXPERIMENTAL  
MEASURED AND ANALYTICAL MODELED NATURAL  
FREQUENCIES

Experimental, Hz	Analytical, Hz	Difference, percent
658	658	0
1049	1006	-4.1
1710	1762	3.0
2000	2051	2.6
2276	2336	2.6
2536	2536	0
2722	2752	1.1
2962	3012	1.7

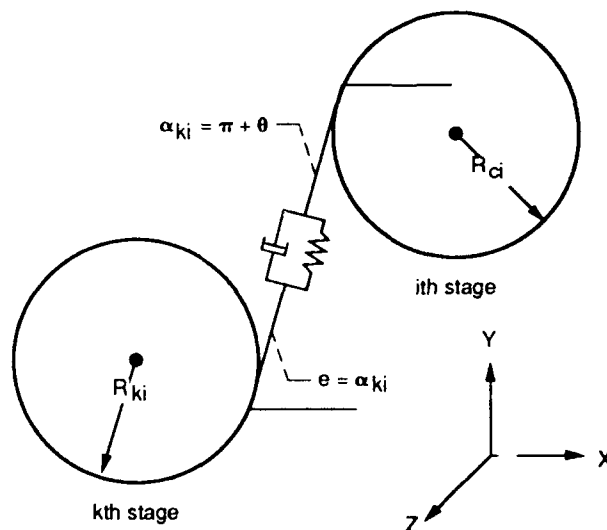


Figure 1.—Geometry of simulation of gear force.

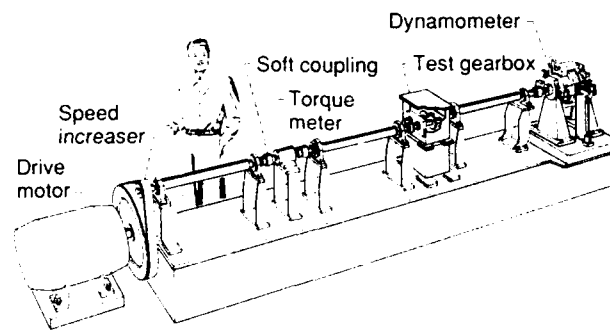


Figure 2.— Picture of gear noise rig.

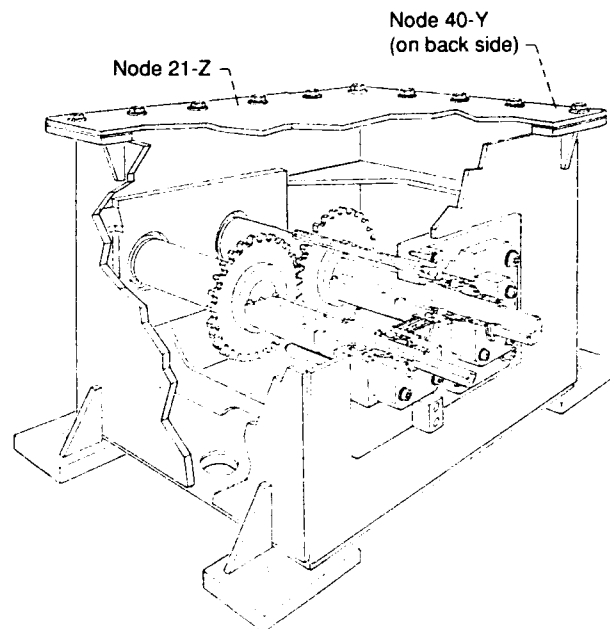


Figure 3.— Test gear box.

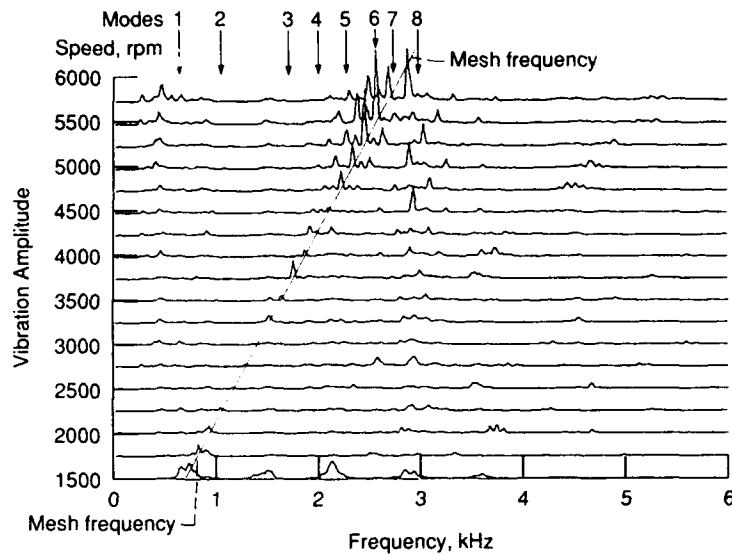


Figure 4.— Experimental vibration frequency spectrum at node 21 (note: The measured modal frequencies are shown at top of figure).

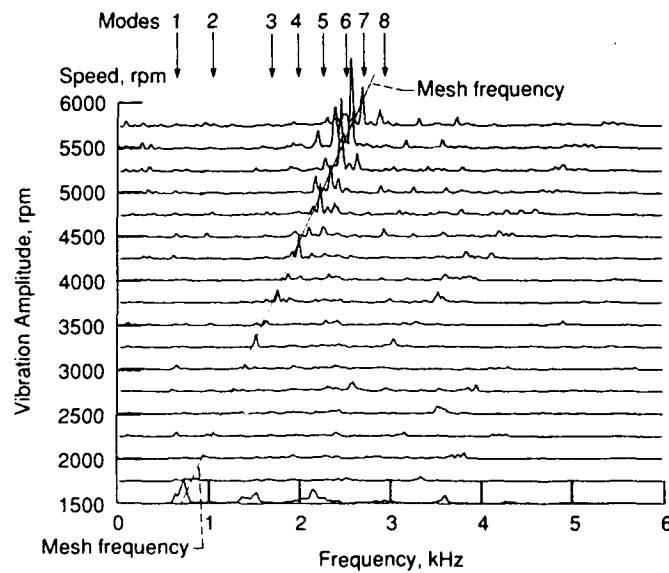
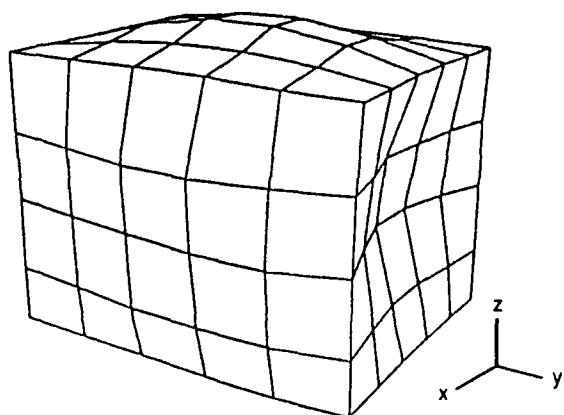
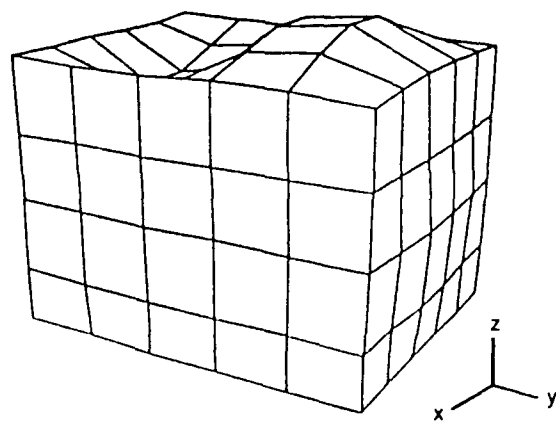


Figure 5.— Experimental vibration frequency spectrum at node 40 (note: The measured modal frequencies are shown at top of figure).

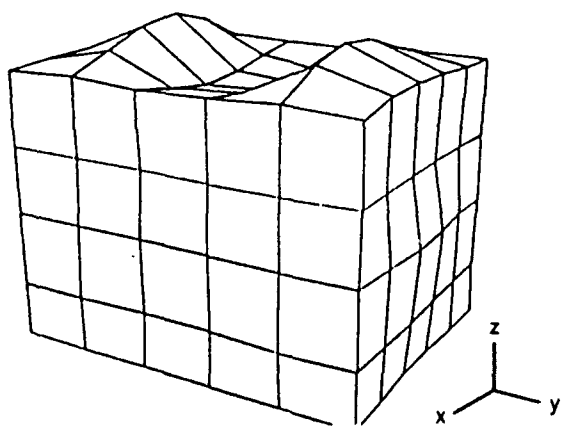




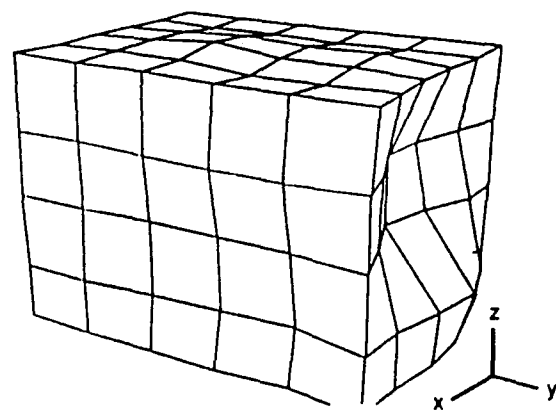
Mode: 1  
Frequency, 658.37 Hz



Mode: 2  
Frequency, 1048.56 Hz

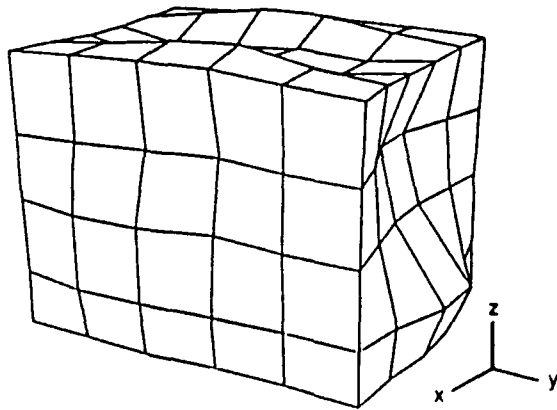


Mode: 3  
Frequency, 1709.77 Hz

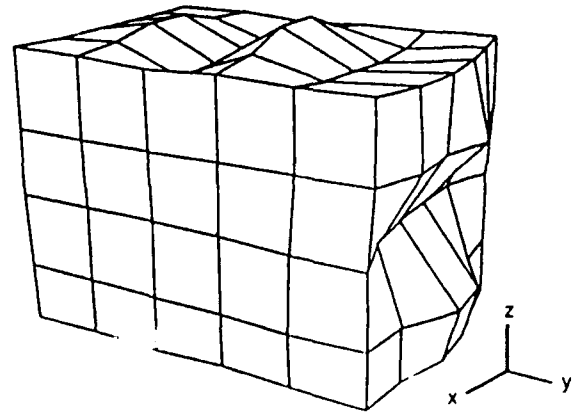


Mode: 4  
Frequency, 1999.95 Hz

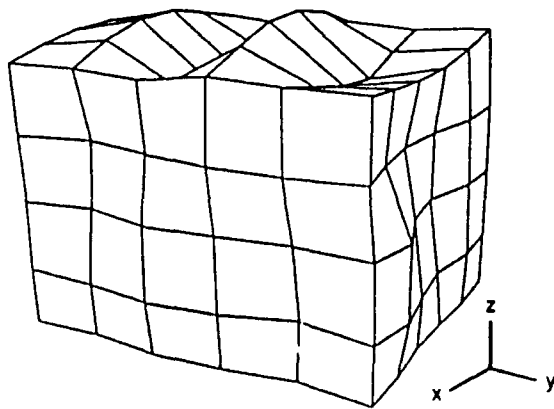
Figure 6.— Gear box experimental mode shapes.



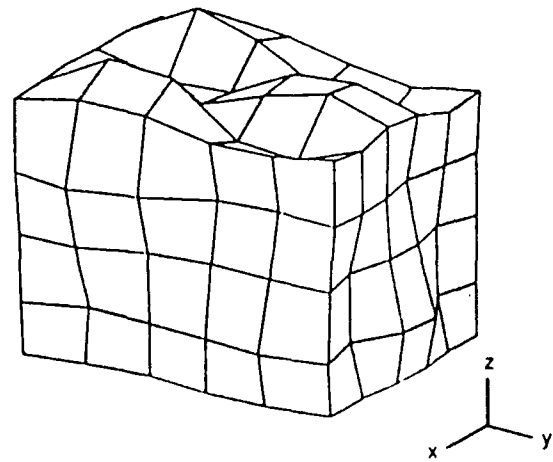
Mode: 5  
Frequency, 2275.69 Hz



Mode: 6  
Frequency, 2535.77 Hz

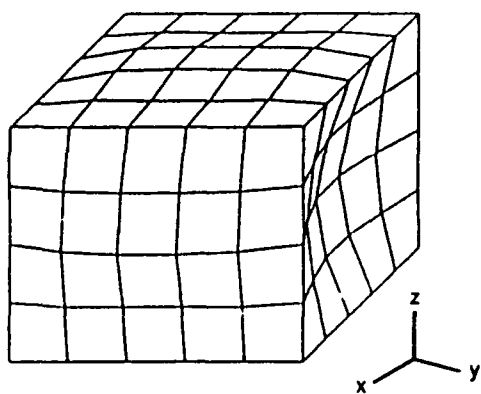


Mode: 7  
Frequency, 2722.16 Hz

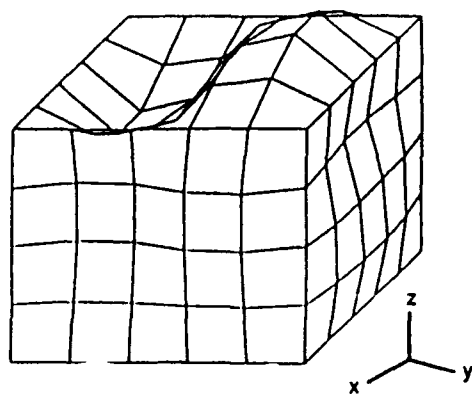


Mode: 8  
Frequency, 2961.71 Hz

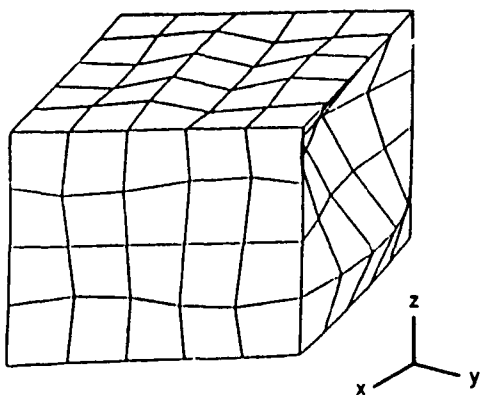
Figure 7.--- Gear box experimental mode shapes.



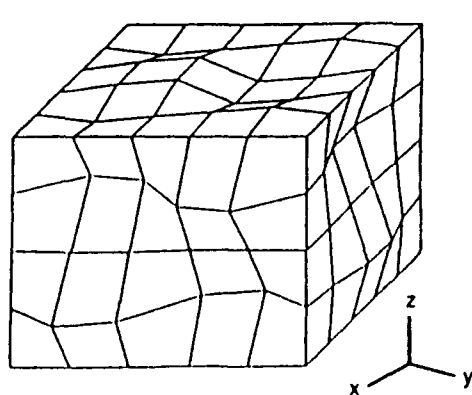
Mode: 1  
Frequency = 658



Mode: 2  
Frequency = 1006

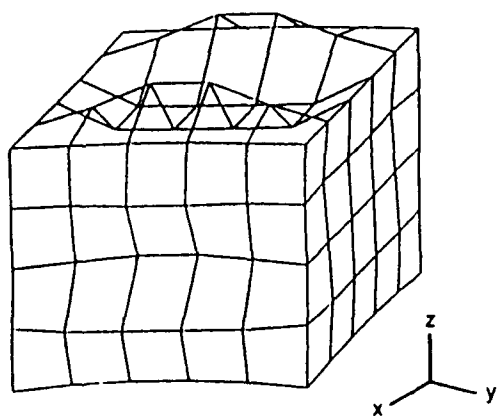


Mode: 3  
Frequency = 1762

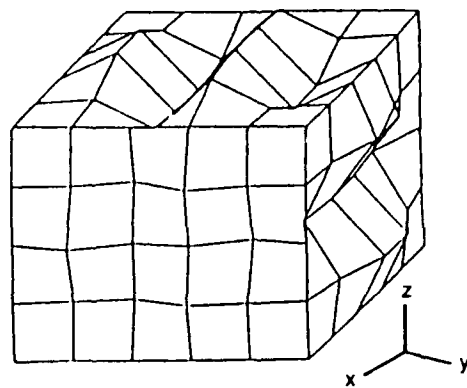


Mode: 4  
Frequency = 2051

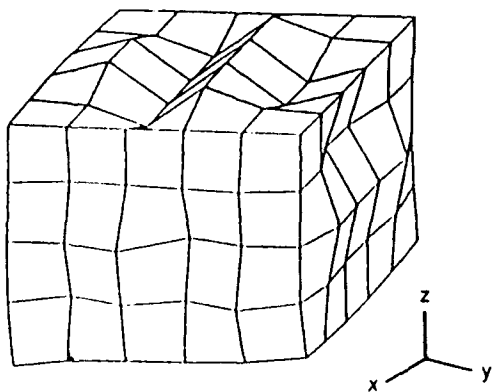
Figure 8.— Gear box analytical mode shapes.



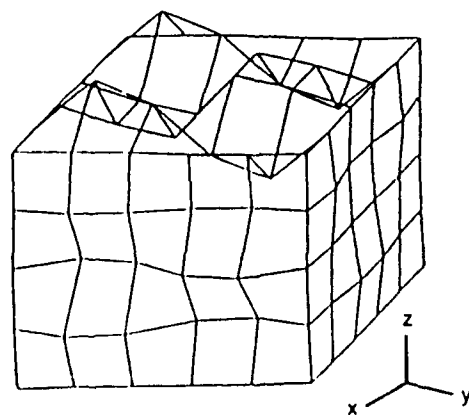
Mode: 5  
Frequency = 2336



Mode: 6  
Frequency = 2536



Mode: 7  
Frequency = 2752



Mode: 8  
Frequency = 3012

Figure 9.— Gear box analytical mode shapes.

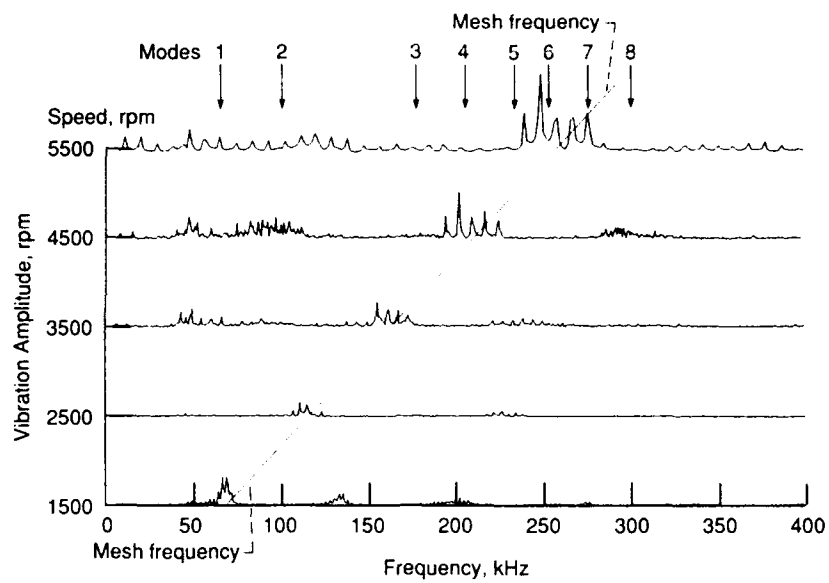


Figure 10.— Analytical vibration frequency spectrum at node 21 (note: The predicted modal frequencies are shown at top of figure).

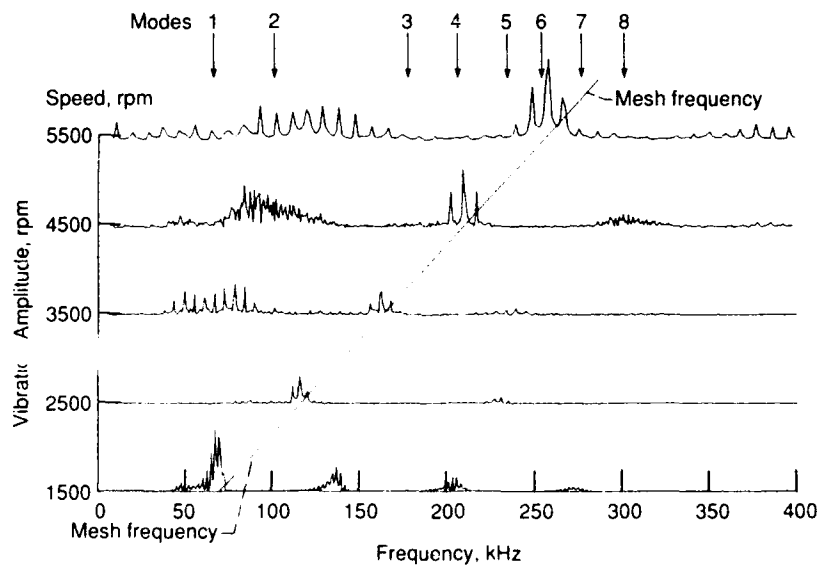


Figure 11.— Analytical vibration frequency spectrum at node 40 (note: The predicted modal frequencies are shown at top of figure).

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16. Abstract This paper presents an analytical simulation of the dynamics of a gear transmission system compared to experimental results from a gear noise test rig at the NASA Lewis Research Center. The analytical procedure developed couples the dynamic behaviors of the rotor-bearing-gear system with the response of the gearbox structure. The modal synthesis method is used in solving the overall dynamics of the system. Locally each rotor-gear stage is modeled as an individual rotor-bearing system using the matrix transfer technique. The dynamics of each individual rotor are coupled with other rotor stages through the nonlinear gear mesh forces and with the gearbox structure through bearing support systems. The modal characteristics of the gearbox structure are evaluated using the finite element procedure. A variable time stepping integration scheme is used to calculate the overall time transient behavior of the system in modal coordinates. The global dynamic behavior of the system is expressed in a generalized coordinate system. Transient and steady state vibrations of the gearbox system are presented in the time and frequency domains. The vibration characteristics of a simple single mesh gear noise test rig is modeled. The numerical simulations are compared to experimental data measured under typical operating conditions. The comparison of system natural frequencies, peak vibration amplitudes, and gear mesh frequencies are generally in good agreement.					
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